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AN ANALYSIS OF TWO-DIMENSIONAL LAMINAR AND TURBULENT COMPRESSIBLE MIXING

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ARO, Inc.

May 1965

ROCKET TEST FACILITY
ARNOLD ENGINEERING DEVELOPMENT CENTER
AIR FORCE SYSTEMS COMMAND
ARNOLD AIR FORCE STATION, TENNESSEE

NOTICES

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FOREWORD

The research reported herein was conducted by ARO, Inc. (a subsidiary of Sverdrup and Parcel, Inc.), contract operator of the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), Arnold Air Force Station, Tennessee, under Contract AF 40(600)-1000, Program Element 62405184/6950, Task 695002. The research was conducted between September 22 and December 14, 1964, under ARO Project No. RW2512, and the report was submitted by the author on March 31, 1965.

This technical report has been reviewed and is approved.

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ABSTRACT

An analysis is presented of two-dimensional, isoenergetic, compressible mixing of a jet with a fluid at rest for both laminar and turbulent mixing. The analysis is primarily concerned with the development of theoretical expressions for the mixing similarity parameters. A general momentum equation is derived, which relates the similarity parameter to the mixing length and viscous shear stress. By using this general equation and Newton's viscous shear relation, a complete theoretical solution for the laminar mixing similarity parameter is derived, which does not involve a reference perturbation velocity factor. Prandtl's mixing length theory for the apparent viscous shear relation is used to obtain the theoretical similarity parameter for turbulent mixing. Based on these similarity parameters, equations for the width of the corresponding mixing regions are derived. These results and the approximate theoretical velocity profile equation developed by Pai, Nash, and Korst represent a closed-form theoretical approximation of the type of mixing considered.

CONTENTS

ILLUSTRATIONS

NOMENCLATURE

b	Width of mixing zone
C_a	Crocco number
$(I_1)_{\eta_D} =$	$\int_{-\eta_M}^{\eta_D} \frac{\phi^2}{1 - C_{a_\infty}^2 \phi^2} d\eta$
k	Ratio of Prandtl's mixing length to the width of the mixing zone
ℓ	Prandtl's mixing length

M	Mach number
n	Exponent of temperature in temperature-viscosity relation
p	Base pressure and static pressure throughout mixing zone and inviscid flow field
R	Gas constant
T	Temperature
u	Velocity
X	Distance along centerline of mixing zone (Fig. 1)
Y	Vertical distance from centerline of mixing zone (Fig. 1)
y	Vertical distance from dividing streamline
α	Reference perturbation velocity factor, u_∞/u_R
γ	Ratio of specific heats
η	Non-dimensional mixing ordinate, $\sigma Y/X$
μ	Viscosity
ν	Kinematic viscosity, μ/ρ
ρ	Density
σ	Similarity parameter
τ	Viscous shear stress
ϕ	Velocity ratio, u/u_∞

SUBSCRIPTS

D	Dividing streamline
M	Extremities of mixing region
O	Incompressible, i.e., corresponding to $C_{a_\infty} = 0$
R	Reference
t	Total
∞	Free stream

SECTION I
INTRODUCTION

The type of mixing considered is two-dimensional, compressible, isobaric, and isoenergetic without an initial boundary layer. The object is to define theoretically, in closed form, the following characteristics for both laminar and turbulent mixing:

a. The velocity profile
b. The width of the mixing region

For laminar mixing, both of these characteristics were numerically obtained for specific cases by Chapman (Ref. 1). An approximate velocity profile for laminar mixing was obtained by Nash (Ref. 2), who used the simplified equation of motion of the heat conduction form, derived by Pai (Ref. 3). However, this equation contains an unknown reference perturbation velocity factor for which Pai assumes a value of unity and Nash assumes a value of 2.0. Nash verifies his assumption by comparing with the numerical values obtained by Chapman. For turbulent mixing, Korst (Ref. 4) also reduced the equation of motion by the method of small perturbations to the heat conduction equation form. Thus, the velocity profile for both laminar and turbulent mixing is approximated by the equation

$$\phi = \frac{1}{2}\left[1 + \operatorname{erf}\eta\right]$$

where the error function

$$\operatorname{erf}\eta = \frac{2}{(\pi)^{1/2}}\int_0^{\eta} e^{-\eta^2}\, d\eta$$

$$\eta = \frac{\sigma Y}{X}$$

The difference between laminar and turbulent mixing is reflected in the relationship for the similarity parameter, σ. For turbulent mixing, experiment shows σ to be a function only of the free-stream Crocco number since the mixing zone width varies linearly with the mixing length, X. For laminar mixing, Page and Dixon (Ref. 5) developed the following expression for σ (which can also be obtained by inference from Nash's work):

$$\sigma = \frac{1}{2}\left(\frac{u_\infty X}{a\,\nu_\infty}\right)^{1/2}$$

where a is the unknown reference perturbation velocity factor. Thus, in general, the similarity parameter, σ, is a function of both the free-stream conditions and the mixing length, X.

The following is a derivation, which yields a complete theoretical relation for the laminar mixing similarity parameter and a semi-theoretical

relation for the turbulent similarity parameter based on Prandtl's mixing length theory.

SECTION II
BASIC EQUATIONS

The purpose of this analysis is to relate the mixing similarity parameter, σ, to the mixing length, X, and the viscous shear stress, τ, in a manner that is applicable to either laminar or turbulent mixing. Such a relationship can be derived by applying the conservation of momentum condition to the control volume shown in the following figure of a general mixing zone.

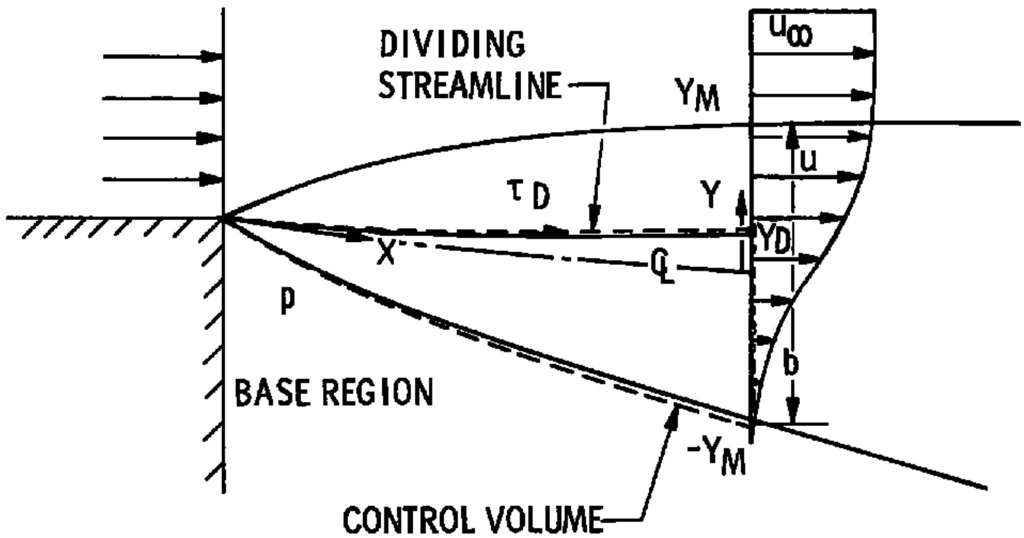

Fig. 1 General Mixing Zone

The following are assumed:

1. Over the mixing zone length, X, considered, the average angle between the dividing streamline and the mixing zone centerline is small (< 20°).
2. The mixing is two-dimensional, isobaric, and isoenergetic with the non-dimensional velocity profile independent of X.

The momentum equation for this control volume is

$$\int_0^X \tau_D \, dX = \int_{-Y_M}^{Y_D} \rho u^2 \, dY \tag{1}$$

By the Perfect Gas law

$$\rho = \frac{p}{RT} = \frac{p}{RT_t(1 - C_{a_\infty}^2 \phi^2)}$$

where

$$\phi = \frac{u}{u_\infty}$$

$$C_{a_\infty}^2 = 1 - \frac{T_\infty}{T_t}$$

let

$$\eta = \sigma \frac{Y}{X}$$

where

σ = similarity parameter and is a function of C_{a_∞} and X.

Substituting into Eq. (1) yields

$$\int_0^X \tau_D \, dX = \left(\frac{p}{RT_t}\right)\left(u_\infty^2\right)\left(\frac{X}{\sigma}\right) \int_{-\eta_M}^{\eta_D} \frac{\phi^2}{1 - C_{a_\infty}^2 \phi^2} \, d\eta \tag{2}$$

Let

$$(I_3)_{\eta_D} = \int_{-\eta_M}^{\eta_D} \frac{\phi^2}{1 - C_{a_\infty}^2 \phi^2} \, d\eta$$

The integral, I_3, has been numerically evaluated in Refs. 6 and 7 based on the non-dimensional approximate velocity profile discussed in Section I. As a consequence of assumption 2, I_3 is independent of X. Therefore, the derivative of Eq. (2) with respect to X is

$$\tau_D = \left[\frac{p \, u_\infty^2 \, (I_3)_{\eta_D}}{RT_t}\right] \frac{d\left(\frac{X}{\sigma}\right)}{dX} \tag{3}$$

Equation (3) is the desired general relation between the viscous shear stress, τ_D, the mixing length, X, and the similarity parameter, σ. Equation (3) also embodies the conservation of the X-direction momentum and mass flow in the transverse Y direction since the location of the dividing streamline is determined by applying these conditions.

2.1 LAMINAR MIXING

The similarity parameter, σ, for laminar mixing can be derived from Eq. (3) in the following manner. For laminar mixing, the viscous shear stress is determined by Newton's relation as follows

$$\tau_D = \mu_D \left(\frac{du}{dY}\right)_D$$

Since

$$\frac{du}{dY} = \left(\frac{du}{d\eta}\right)\left(\frac{d\eta}{dY}\right) = u_\infty \left(\frac{d\phi}{d\eta}\right)\left(\frac{d\eta}{dY}\right)$$

and

$$\frac{d\eta}{dY} = \frac{\sigma}{X}$$

therefore

$$\tau_D = u_\infty \mu_D \left(\frac{\sigma}{X}\right)\left(\frac{d\phi}{d\eta}\right)_D \tag{4}$$

Substituting Eq. (4) into Eq. (3) yields

$$\mu_D \left(\frac{d\phi}{d\eta}\right)_D \left(\frac{\sigma}{X}\right) = \left[\frac{p\,u_\infty}{RT_t} (I_3)_{\eta_D}\right] \frac{d\left(\frac{X}{\sigma}\right)}{dX}$$

Separating the variables and integrating yields

$$\mu_D \left(\frac{d\phi}{d\eta}\right)_D X = \frac{1}{2}\left[\frac{p\,u_\infty}{RT_t} (I_3)_{\eta_D}\right]\left(\frac{X}{\sigma}\right)^2$$

Solving for σ gives

$$\sigma = \left(\frac{p\,u_\infty (I_3)_{\eta_D} X}{2RT_t \mu_D \left(\frac{d\phi}{d\eta}\right)_D}\right)^{1/2} \tag{5}$$

The width (b) of a laminar mixing zone can be determined from the definition of η and Eq. (5) as follows since

$$\eta_M = \frac{\sigma b}{2X}$$

$$\therefore\ b = 2\eta_M \frac{X}{\sigma}$$

Substituting Eq. (5) for σ yields

$$b = 2\,\eta_M \left[\frac{2\,RT_t\,\mu_D \left(\frac{d\phi}{d\eta}\right)_D X}{p\,u_\infty\,(I_3)_{\eta_D}}\right]^{1/2} \tag{6}$$

Thus, for given free-stream conditions, the width of a laminar mixing zone varies linearly with $X^{1/2}$. The value of η_M is determined when the limits of the mixing zone are defined. A mixing zone is usually defined to be that region which exists between $\eta_M = \pm 1.503$, which corresponds to velocity ratios, ϕ, of 0.9845 and 0.0154 based on the approximate velocity profile equation. This definition is based on Tollmien's results in Ref. 7.

From Eq. (5), the η ordinate for laminar mixing is

$$\eta = \sigma\,\frac{Y}{X} = Y\left[\frac{p\,u_\infty (I_3)_{\eta_D}}{2RT_t\,\mu_D \left(\frac{d\phi}{d\eta}\right)_D X}\right]^{1/2} \tag{7}$$

since

$$\rho_\infty = \frac{p}{RT_\infty}$$

$$\frac{\mu_D}{\mu_\infty} = \left(\frac{T_D}{T_\infty}\right)^n$$

Substituting into Eq. (7) yields

$$\eta = \left[\frac{(I_3)_{\eta_D}\left(1 - C_{a_\infty}^2\right)^{n+1}}{2\left(\frac{d\phi}{d\eta}\right)_D\left(1 - C_{a_\infty}^2\,\phi_D^2\right)^n}\right]^{1/2} (Y) \left(\frac{u_\infty}{\nu_\infty X}\right)^{1/2} \tag{8}$$

In Ref. 1 velocity profiles for laminar mixing are presented as a function of $y\left(\frac{u_\infty}{\nu_\infty X}\right)^{1/2}$ and are shown for various free-stream Mach numbers in the range from 0 to 5.0. The velocity profiles for free-stream Mach numbers of 0 and 5.0 have been replotted in Fig. 2 as a function of η (Eq. 8) using the compressible value of $(I_3)_{\eta_D}$ from Ref. 8 and $\left(\frac{d\phi}{d\eta}\right)_D$ evaluated from the approximate mixing velocity profile also presented in Fig. 2.

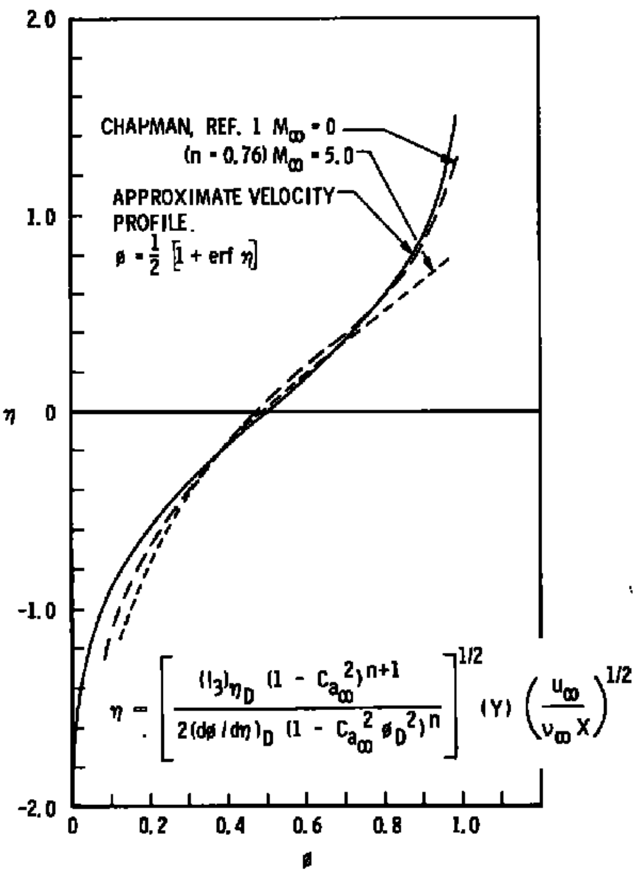

Fig. 2 Correlation of Laminar Mixing Velocity Profiles

The parameters were evaluated at the dividing streamline location shown in Ref. 1 to correspond to $\phi_D = 0.61$ for $n = 0.76$. Figure 2 shows that the velocity profiles for laminar mixing are correlated very well by the η variable defined by Eq. (8). The deviation of the exact velocity profile for $M = 5.0$ from the approximate theoretical profile is due to the fact that, for laminar mixing, the exact velocity profile is a weak function of the free-stream Mach number. This is also indicated by the invariance of the dividing streamline velocity ratio with free-stream Mach number.

In Ref. 5 for laminar mixing, the following relation for σ is presented

$$\sigma = \frac{1}{2}\left(\frac{u_\infty X}{\nu_\infty a}\right)^{1/2} \tag{9}$$

where a is a reference perturbation velocity factor. A theoretical relation for a can be obtained from Eq. (5) in the following manner. Eq. (5) can be written as follows:

$$\sigma = \left[\frac{(I_3)_{\eta_D}\left(1 - C_{a_\infty}^2\right)^{n+1}}{2\left(\frac{d\varphi}{d\eta}\right)_D\left(1 - C_{a_\infty}^2 \phi_D^2\right)^n}\right]^{1/2}\left(\frac{u_\infty X}{\nu_\infty}\right)^{1/2} \tag{10}$$

Equating Eqs. (9) and (10) and solving for a yields

$$a = \frac{\left(\frac{d\phi}{d\eta}\right)_D\left(1 - C_{a_\infty}^2 \phi_D^2\right)^n}{2\,(I_3)_{\eta_D}\left(1 - C_{a_\infty}^2\right)^{n+1}} \tag{11}$$

The theoretical variation of a with free-stream Mach number is presented in Fig. 3. The parameter, a, is shown in Fig. 3 to have a value of 2.112 for $C_{a_\infty}^2 = 0$, which agrees with the value of 2.0 assumed by Nash (Ref. 2). However, a increases in value as the free-stream Mach number increases.

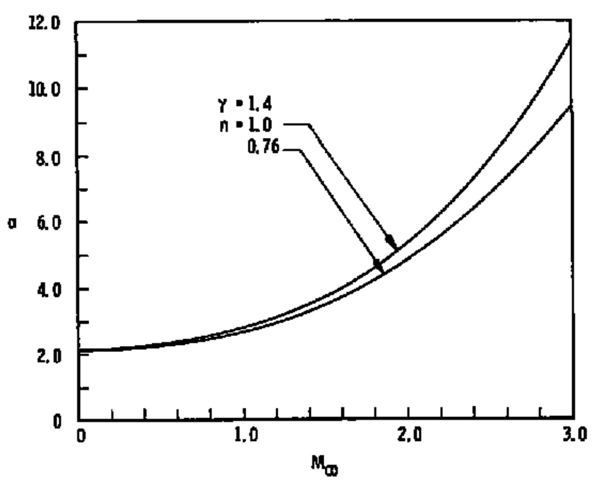

Fig. 3 Theoretical Variation of the "Reference Perturbation Velocity Factor"

2.2 TURBULENT MIXING

The similarity parameter, σ, for turbulent mixing can be derived from Eq. (3) in the following manner. For turbulent mixing, the apparent viscous shear stress can be defined by Prandtl's mixing length theory

$$\tau_D = \rho_D \ell^2 \left(\frac{du}{dY}\right)_D^2$$

or

$$\tau_D = \frac{\rho_D u_\infty^2 \ell^2 \sigma^2}{X^2} \left(\frac{d\phi}{d\eta}\right)_D^2 \qquad (12)$$

Since the mixing length, ℓ, is linearly related to the effective vortex size, then the following relation for ℓ is consistent with assumption (2).

$$\ell = kb$$

where

$$k = \text{non-dimensional unknown constant}$$

By definition

$$\eta_M = \frac{\sigma b}{2X}$$

or

$$b = 2\eta_M \left(\frac{X}{\sigma}\right)$$

$$\therefore \ell = 2\eta_M k \left(\frac{X}{\sigma}\right)$$

Since ℓ varies linearly with the ratio, X/σ (not X as assumed by Tollmien and others), the effect of compressibility is taken into account and the constant, k, will be independent of the free-stream Mach number. Substituting into Eq. (12) yields

$$\tau_D = 4\eta_M^2 k^2 \rho_D u_\infty^2 \left(\frac{d\phi}{d\eta}\right)_D^2$$

$$\rho_D = \frac{p}{RT_D} = \frac{p}{RT_t\left(1 - C_{a_\infty}^2 \phi_D^2\right)}$$

$$\therefore \tau_D = \frac{4\eta_M^2 k^2 p u_\infty^2}{RT_t\left(1 - C_{a_\infty}^2 \phi_D^2\right)} \left(\frac{d\phi}{d\eta}\right)_D^2 \qquad (13)$$

Substituting Eq. (13) into Eq. (3) yields

$$\frac{4\eta_M^2 k^2}{\left(1 - C_{a_\infty}^2 \phi_D^2\right)} \left(\frac{d\phi}{d\eta}\right)^2 = (I_3)_{\eta_D} \frac{d\left(\frac{X}{\sigma}\right)}{dX}$$

Separating the variables and integrating yields

$$\sigma = \frac{(I_3)_{\eta_D}\left(1 - C_{a_\infty}^2 \phi_D^2\right)}{4\eta_M^2 k^2 \left(\frac{d\phi}{d\eta}\right)_D^2} \qquad (14)$$

Tollmien (Ref. 7) experimentally determined a value of 12 for σ from low subsonic, $C_{a_\infty}^2 \approx 0$, turbulent mixing experiments. Based on this result and using the approximate velocity profile and the theoretical dividing streamline location from Ref. 8, the value of the constant, k, in Eq. (14) is 0.06895 for $\eta_M = 1.503$. Substituting these values into Eq. (14) yields

$$\sigma = \frac{(I_3)_{\eta_D}\left(1 - C_{a_\infty}^2 \phi_D^2\right)}{(0.04235)\left(\frac{d\phi}{d\eta}\right)_D^2} \tag{15}$$

The equation for σ relative to the incompressible value, $C_{a_\infty} = 0$, is

$$\frac{\sigma}{\sigma_0} = \frac{(I_3)_{\eta_D}\left(1 - C_{a_\infty}^2 \phi_D^2\right)}{(0.5085)\left(\frac{d\phi}{d\eta}\right)_D^2} \tag{16}$$

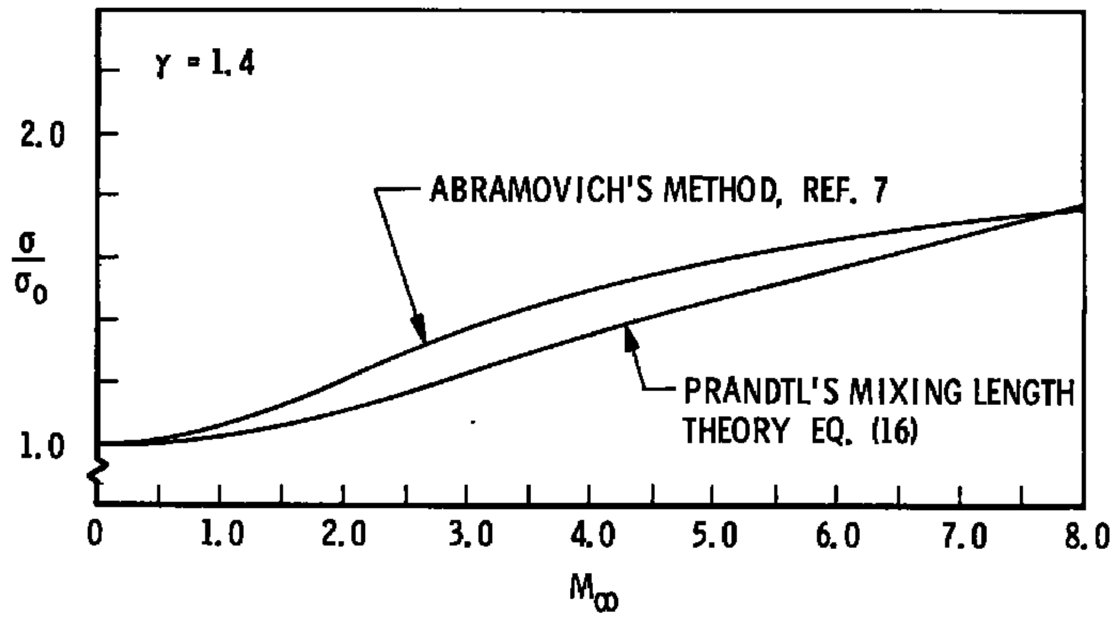

Fig. 4 Compressibility Effect on the Similarity Parameter for Turbulent Mixing

In Fig. 4, Eq. (16) is graphically compared with results using Abramovich's method (numerically evaluated in Ref. 8). The lack of reliable experimental data on this subject precludes any conclusions as to the validity of Eq. (16) or Abramovich's method. However, in Ref. 9, Abramovich's method is indirectly shown to be in general agreement with experiment.

The width, b, of a turbulent mixing zone can be derived in the same manner as for the laminar mixing case. The basic equation is

$$b = 2\,\eta_M \left(\frac{X}{\sigma}\right) \tag{17}$$

Substituting Eq. (15) into Eq. (17) yields

$$b = \frac{(0.0847)\ \eta_M \left(\frac{d\phi}{d\eta}\right)_D^2 X}{(I_3)_{\eta_D}\left(1 - C_{a_\infty}^2\ \phi_D^2\right)} \tag{18}$$

The mixing zone width based on the usual definition of the mixing region, $\eta_M = 1.503$, is

$$b = \frac{(0.1273)\left(\frac{d\phi}{d\eta}\right)_D^2 X}{(I_3)_{\eta_D}\left(1 - C_{a_\infty}\ \phi_D^2\right)} \tag{19}$$

Thus, for given free-stream conditions, the width of a turbulent mixing zone varies linearly with X. The η ordinate for turbulent mixing is

$$\eta = \frac{(I_3)_{\eta_D}\left(1 - C_{a_\infty}^2\ \phi_D^2\right)}{(0.04235)\left(\frac{d\phi}{d\eta}\right)_D^2}\left(\frac{Y}{X}\right) \tag{20}$$

SECTION III
CONCLUSIONS

The velocity profile for laminar or turbulent two-dimensional mixing is approximated very well by the equation

$$\phi = \frac{1}{2}[1 + \operatorname{erf}\ \eta] \tag{21}$$

where

$$\eta = \frac{\sigma Y}{X}$$

Within the framework of the assumptions of this analysis, the similarity parameter, σ, for laminar mixing is

$$\sigma = \left[\frac{(I_3)_{\eta_D}\left(1 - C_{a_\infty}^2\right)^{n+1} u_\infty X}{2\left(\frac{d\phi}{d\eta}\right)_D \left(1 - C_{a_\infty}^2\ \phi_D^2\right)^n \nu_\infty}\right]^{1/2} \tag{22}$$

The location of the dividing streamline for laminar mixing has been theoretically determined by Chapman and is independent of free-stream velocity. Thus, the laminar mixing problem is theoretically defined, in closed form, by Eqs. (21) and (22) and Chapman's results without the need of a reference perturbation velocity factor, a.

The similarity parameter for turbulent mixing based on Prandtl's mixing length theory is

$$\sigma = \frac{(I_3)_{\eta_D}\left(1 - C_{a_\infty}^2 \phi_D^2\right)}{(0.04235)\left(\frac{d\phi}{d\eta}\right)_D^2} \tag{23}$$

The numerical constant was determined from the experimental results obtained by Tollmien. However, this constant is independent of the definition of the mixing region and, based on the agreement with Abramovich's method, also independent of the free-stream velocity. These results confirm the applicability of Prandtl's mixing theory to this type of compressible mixing.

The similarity parameters for both laminar and turbulent mixing were derived from a single differential equation, which relates σ to the mixing length, X, and the viscous shear stress on the dividing streamline, τ_D. This equation is

$$\tau_D = \left[\frac{\rho\, u_\infty^2\, (I_3)_{\eta_D}}{R T_t}\right] \frac{d\left(\frac{X}{\sigma}\right)}{dX} \tag{24}$$

Although this is an analysis of a relatively simple mixing zone, the approach can be applied to more complex mixing processes such as two-dimensional or axisymmetric, non-isoenergetic mixing with or without a secondary stream.

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It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, or *(U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.